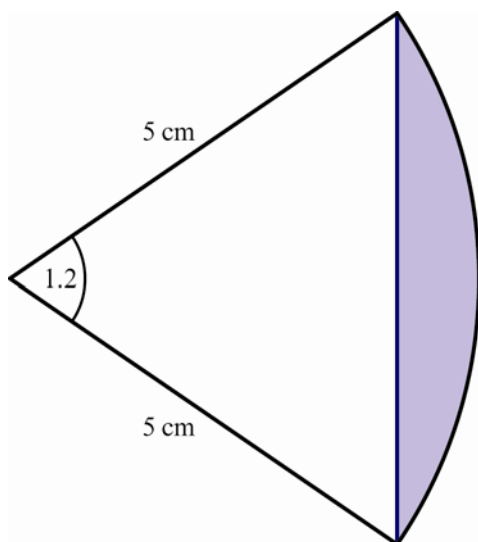


**Revision: Geometry (Topics 3 & 4)****Coursebook chapters: 9–14**

1. Find the area of the shaded region.



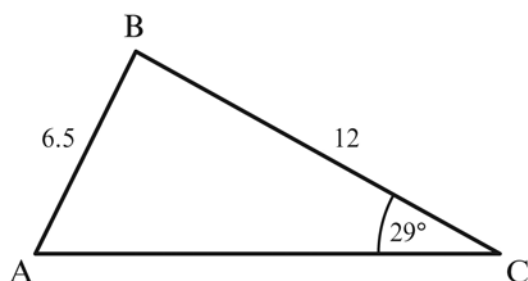
*(accessible to students on the path to grade 3 or 4) [4 marks]*

2. Do not use a calculator to answer this question.

Find the exact solutions of the equation  $\sin 2\theta = \sqrt{3} \cos 2\theta$  for  $0 \leq \theta \leq 180^\circ$ .

*(accessible to students on the path to grade 3 or 4) [5 marks]*

3. In triangle ABC,  $AB = 6.5$  cm,  $BC = 12$  cm and  $\hat{C} = 29^\circ$ .



Find the two possible values of  $\hat{A}$ .

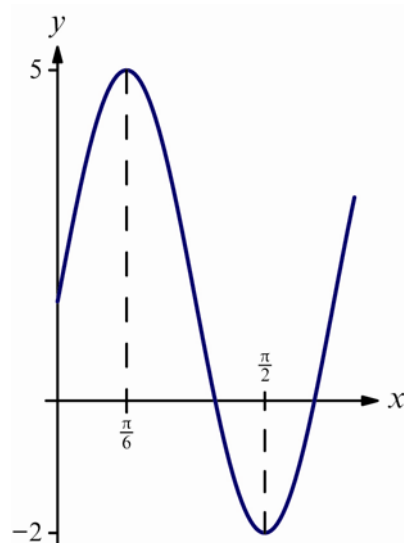
*(accessible to students on the path to grade 3 or 4) [5 marks]*

4. Find the angles of the triangle with vertices  $A(4, 6, 2)$ ,  $B(1, 1, 2)$  and  $C(0, -1, 3)$ . Give your answers correct to the nearest degree.

*(accessible to students on the path to grade 3 or 4) [9 marks]*

5. Do not use a calculator to answer this question.

The graph of the function  $f(x) = A \sin(kx) + B$  is shown below.



Find the values of  $k$ ,  $A$  and  $B$ .

*(accessible to students on the path to grade 3 or 4) [5 marks]*

6. Do not use a calculator to answer this question.

Solve the equation  $\cos(2x + 30^\circ) = \frac{1}{2}$  for  $0^\circ \leq x \leq 360^\circ$ .

*(accessible to students on the path to grade 5 or 6) [5 marks]*

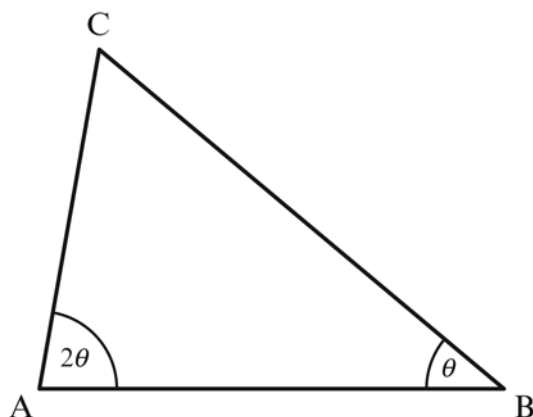
7. Find the coordinates of the point where the line with Cartesian equation  $\frac{x-3}{5} = \frac{y+1}{3} = z-4$  intersects the plane with equation  $4x - y + 2z = 7$ .

*(accessible to students on the path to grade 5 or 6) [5 marks]*

**8 Do not use a calculator to answer this question.**

Solve the equation  $\tan^2 x - \sec x - 1 = 0$  for  $x \in [0, 2\pi]$ .

*(accessible to students on the path to grade 5 or 6) [7 marks]*

**9. Triangle ABC has  $\hat{A} = 2\theta$ ,  $\hat{B} = \theta$  and  $BC : AC = 5 : 4$ .**

- (a) Find the exact value of  $\cos \theta$ .
- (b) Hence find the angles of the triangle.

*(accessible to students on the path to grade 5 or 6) [6 marks]*

**10. Do not use a calculator to answer this question.**

- (a) Express  $\sqrt{3} \cos x - \sin x$  in the form  $R \cos(x + \theta)$  where  $R > 0$  and  $\theta \in \left[0, \frac{\pi}{2}\right]$ .

- (b) State the maximum value of  $\frac{3}{5 + \sqrt{3} \cos x - \sin x}$ .

*(accessible to students on the path to grade 5 or 6) [6 marks]*

**11. Three planes have equations  $4x - y + z = 8$ ,  $x + y - 2z = 3$  and  $6x + y - 3z = a$ .**

- (a) Find the value of  $a$  for which the intersection of the three planes is a straight line.
- (b) Find the equation of this line.

*(accessible to students on the path to grade 5 or 6) [8 marks]*

12. Line  $l_1$  has equation  $\mathbf{r} = (6\mathbf{i} - 7\mathbf{j} - 7\mathbf{k}) + s(2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k})$ . Line  $l_2$  passes through the origin and the point  $B(4, -1, -2)$ . The two lines intersect at point A.

(a) Write down a vector equation of  $l_2$ .

(b) Find the coordinates of A.

(c) Find the angle between  $l_1$  and  $l_2$ .

*(accessible to students on the path to grade 3 or 4)*

(d) Find the distance AB, and hence find the perpendicular distance from B to  $l_1$ .

*(accessible to students on the path to grade 5 or 6)*

*[12 marks]*

13. Given that  $A = \arcsin\left(\frac{1}{3}\right)$  find the exact value of  $\cos A$ .

*(accessible to students on the path to grade 7) [3 marks]*

14. Do not use a calculator to answer this question.

Given that  $\theta \in \left(0, \frac{\pi}{2}\right)$  and that vectors  $\begin{pmatrix} \cos \theta \\ \sin \theta \\ -\sin \theta \end{pmatrix}$  and  $\begin{pmatrix} \cos \theta \\ -\sin \theta \\ \cos \theta \end{pmatrix}$  are perpendicular,

(a) Show that  $\tan 2\theta = 2$ .

(b) Hence find the exact value of  $\tan \theta$ .

*(accessible to students on the path to grade 7) [9 marks]*

15. (a) Use the formulae for  $\sin(A + B)$  and  $\sin(A - B)$  to show that:  
 $\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$ .

- (b) Hence show that  $\sin \theta + \sin 3\theta = 4 \sin \theta \cos^2 \theta$ .

*(accessible to students on the path to grade 7) [5 marks]*

16. (a) Find the angle between the lines  $\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} + t \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$  and  $\mathbf{r} = \begin{pmatrix} k \\ 2 \\ -3 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ .

*(accessible to students on the path to grade 3 or 4)*

- (b) Find the value of  $k$  for which the lines  $l_1$  and  $l_2$  intersect.

*(accessible to students on the path to grade 5 or 6)*

- (c) (i) Calculate  $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ .

- (ii) For the value of  $k$  found in part (b), find the Cartesian equation of the plane  $\Pi$  containing lines  $l_1$  and  $l_2$ .

- (d) Find the exact distance of the point  $(5, 1, 2)$  from  $\Pi$ .

*(accessible to students on the path to grade 7)[15 marks]*